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LETTERS SECTION

Comment on Kálnay's Note on Fundamental Fields

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In a recent interesting note by Kálnay (1975), he argued that the nonlinear scalar field operator, ϕ, for elementary particles, obeying the proportionality

$$\Box\phi \propto (\phi^*\phi)\phi \tag{1}$$

is a more fundamental starting point in a quantum field theory of elementary particles than Heisenberg's nonlinear spinor field relation

$$\gamma^\mu \partial_\mu \psi \propto (\bar{\psi}\psi)\psi \tag{2}$$

The reason given was (as he showed earlier) that "quantum Fermi fields can be entirely described in terms of c numbers and of Bose fields acting on Bose states."

I should like to point out a very basic reason for starting with a spinor formalism rather than the scalar formalism, in a theory of elementary particles according to Heisenberg's view. It is the requirement of maintaining full compatibility with the most general symmetry requirements of the theory of special relativity. If the most fundamental description of elementary particles is to be in terms of a quantum (or c-number) field theory, satisfying the covariance requirements of the theory of special relativity, then the field operators (or c-number field solutions) for these fundamental particles are the basis functions for the *irreducible* representations of the Poincaré group. This result is strictly a consequence of the symmetry imposed by the theory of special relativity—it is independent of whether the underlying symmetry group is to relate to the covariance of a quantum or a c-number field theory!

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It was discovered by Einstein and Mayer (1932), not too long after Dirac's discovery of the electron equation, that the *irreducible* representations of the Poincaré group—the group of only continuous transformations that leave invariant the squared interval, $ds^2 = (dx_0)^2 - (dr)^2$—have the two-dimensional Hermitian structure of quaternions, and the basis functions of these representations are the two-component spinor variables. Thus, the symmetry imposed by the theory of special relativity implies that the basic building blocks for elementary matter must necessarily be in terms of the two-component spinor field variables. (See also Sachs, 1970.)

The building up of spinor field variables of higher rank, in terms of multi-rank spinors, scalar, vector, or (any-rank) tensor fields, or the building up of the Dirac bispinor (as used in Heisenberg's theory), is then a matter of setting up the basis functions for the higher-dimensional representations, *constructed from* the (most) primitive two-dimensional Hermitian representations of the Poincaré group. From this view, all elementary particle fields must be either the two-component spinor particle fields themselves, or composites of them.

References

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